the underlying equations are combined and modified into a matrix formulation that is amenable to a least-squares solution.

The value of the inner product calculated from the measured difference between the times of arrival of the signal at the two antennas on each baseline specifies a circle in the sky upon which nominally lies the apparent point in the sky from which the signal came. Thus, from the time-of-arrival measurements on all three baselines of the Y-shaped array, it is possible to specify three circles in the sky, all of which nominally contain the apparent point in the sky from which the signal came. Nominally, all three circles would intersect at a single point in the sky corresponding to the direction of arrival. In practice, random measurement errors prevent the three circles from intersecting at a single point; instead, they intersect to define a small trianglelike patch of sky. The least-squares-error solution corresponds to a point near the triangle, such that sum of squares of distances between the solution point and each circle in the sky is the least possible value.

This algorithm has been verified on both synthetic data and measurement data recorded by a prototype short-baseline LDAR system. An analysis of errors revealed that the azimuth error depends only on elevation and that the elevation error is small except near the horizon. Further analysis showed that the addition of a vertical baseline (two additional antennas mounted on the top and bottom of a tower) would add little of value to the measurements and calculations since the LDAR source points are typically above 20° elevation.

The primary source of error in the algorithm is the simplifying assumption that the signal originates at an infinite distance. While this assumption is never strictly true, it provides an acceptable approximation as long as the distance of the signal source is much greater than the length of the baselines. The least-squares approach also reduces this error.

An additional equation that takes account of the curvature of a wavefront arriving from a source at a finite distance has been derived and found to be accurate at close range. An algorithm that would solve iteratively for azimuth, elevation, and range of the source has been proposed but not tested.

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Self-Supervised Dynamical Systems

Mathematical models describe coupled motor and mental dynamics.

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Some progress has been made in a continuing effort to develop mathematical models of the behaviors of multi-agent systems known in biology, economics, and sociology (e.g., systems ranging from single or a few biomolecules to many interacting higher organisms). This effort at an earlier stage was reported in "Characteristics of Dynamics of Intelligent Systems" (NPO-21037), *NASA Tech Briefs*, Vol. 26, No. 12 (December 2002), page 48.

To recapitulate from the cited prior article: Living systems can be characterized by nonlinear evolution of probability distributions over different possible choices of the next steps in their motions. One of the main challenges in mathematical modeling of living systems is to distinguish between random walks of purely physical origin (for instance, Brownian motions) and those of biological origin. Following a line of reasoning from prior research, it has been assumed, in the present development, that a biological random walk can be represented by a nonlinear mathematical model that represents coupled mental and motor dynamics incorporating the psychological concept of reflection or self-image. The nonlinear dynamics impart the lifelike ability to behave in ways and to exhibit patterns that depart from thermodynamic equilibrium. Reflection or self-image has traditionally been recognized as a basic element of intelligence.

The nonlinear mathematical models of the present development are denoted self-supervised dynamical systems. They include (1) equations of classical dynamics, including random components caused by uncertainties in initial conditions and by Langevin forces, coupled with (2) the corresponding Liouville or Fokker-Planck equations that describe the evolutions of probability densities that represent the uncertainties. The coupling is effected by fictitious information-based forces, denoted supervising forces, composed of probability densities and functionals thereof.

The equations of classical mechanics represent motor dynamics — that is, dynamics in the traditional sense, signifying Newton's equations of motion. The evolution of the probability densities represents mental dynamics or self-image. Then the interaction between the physical and metal aspects of a monad is implemented by feedback from mental to motor dynamics, as represented by the aforementioned fictitious forces. This feedback is what makes the evolution of probability densities nonlinear. The deviation from linear evolution can be characterized, in a sense, as an expression of free will.

It has been demonstrated that probability densities can approach prescribed attractors while exhibiting such patterns as shock waves, solitons, and chaos in probability space. The concept of self-supervised dynamical systems has been considered for application to diverse phenomena, including information-based neural networks, cooperation, competition, deception, games, and control of chaos. In addition, a formal similarity between the mathematical structures of self-supervised dynamical systems and of quantum-mechanical systems has been investigated.

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